Spanning Trees
Lesson Plan: Class 07 / DM / 02

Overall goal of the lesson	**Recap of Spanning Trees, more exercises, and building cost-effective spanning trees**
Prior knowledge required	Spanning Trees

MODULE 1: **Module time:** 35 minutes

Goal:	More examples of spanning trees; using spanning trees to build cost-effective/cheaper (minimal) spanning trees
Description:	Recap what spanning trees are and see an advanced use for them
Material required:	**Physical:** Writing material, printouts of the work sheet **Electronic:** None
Procedure Details:	1. Slide 2: In the previous class, we had seen an introduction to spanning trees. In this class, we will start by recalling what spanning trees are. We will then explain a slightly advanced use of spanning trees. In this slide, recap the kind of diagrams we are interested in i.e. those with dots and lines connecting them. Give real-life examples to help understand how these abstract diagram could be used in n real-life. E.g. telephone networks, road networks etc. 2. Slide 3: Introduce the concept of 'loops' with the help of an example. Show the three lines in blue and point out how if you start at one dot with a blue line on one end, you can come back to it in a cycle. Run the exercise: Ask them to first see how many loops there are. Then see which lines can be removed such that there are no loops left. Note that there can be more than one correct answer to this puzzle. 3. Slide 4: Show 2 possible answers to the Problem. 4. Slide 5: We'll now move to recalling the concept of the Spanning Tree. Read out the slide lines. Point out that we use the word 'spanning' because such trees 'span' (or cover) all dots. 5. Slide 6: Run the next exercise. 6. Slide 7: Here are the answers. Discuss each of them: 1. No – because one dot is not connected 2. Yes – there are no loops and all dots are 'spanned' 3. Yes 4. No –while all dots are 'spanned' but there is still a loop 7. Slide 8: In this slide, we will recap how spanning trees can help us with real-life problems such as connecting cities with roads. As an example, show the 5 cities which are to be connected. Here, cities are shown as dots and roads between cities will be shown by lines. *(show animation).* Point out how we can connect all cities with just 4 roads instead of connecting each city with each other. 8. Slide 9: Now let's go ahead to more advanced uses of spanning trees. Think about the city map we just made roads for. Mention how in real-life, all roads are not the same: some distances between cities are longer and some are shorter. *Show the animation*.

For this discussion, we will take some dummy distances between these cities (shown here in blue)

9. Slide 9*:* Mention that there is an algorithm for building spanning trees from a given diagram. (If students are unfamiliar with the word 'algorithm', quickly define it as *"a series of steps to solve a problem").*

   We won't be discussing this algorithm in this class, but we will show the result of applying it to build the cheapest road network between cities.

   The selected roads are shown in green. Show that there is at least one route from every city to each other, and that there are no loops.

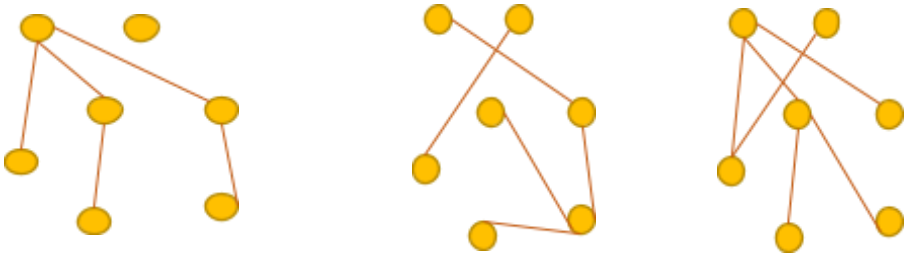
   The total cost of this spanning tree can be found by adding the distances for all the selected (i.e. green) roads. In this case, it is 40.
10. Slide 11: Let's take a problem. Ask students to just add up the route values and compare.
11. Slide 12: Explain the answer. Option 2 is the cheapest because its total cost is 12.
12. Slide 13: The next problem. Students have to make the cheapest spanning tree from this. Explain that this means:
    - Remove all loops
    - Make sure all dots are connected by at least one line
    - There can be many kinds of spanning trees – but pick the one that has the least total cost
13. Slide 13: Here's the answer. Point out that the line with value 7 cannot be chosen because it would have formed a loop with the lines with values 5 and 6 respectively.
14. Conclude: "Today, we recalled with trees and spanning trees are. We then saw a more advanced use of spanning trees to help find the cheapest spanning trees."
15. Give out the worksheet.

ANSWERS TO THE WORKSHEET PROBLEMS

Puzzle 1:

There are many possible solutions – any solution which removes loops is correct.
To check if a solution is correct, see if you can trace a path from one dot back to itself such that you follow a cycle (you go in one direction and return from another)

Here are some examples of correct solutions:



Puzzle 2:

Correct Options are (2) and (4).

(1) is not correct because it has a loop
(3) is not correct because one dot (the rightmost) is not connected to any other dot

Puzzle 3:

For each option, add the numbers associated with each line to come up with the cost of the tree.
Option 1: 2+5+1+4 = 12
Option 2: 3+1+4+3 = 11
Option 3: 2+1+4+3 = 10
Option 4: 2+5+1+3 = 11
Option 3 is the cheapest of these 4 options.

Puzzle 4: The answer is below: