Permutation And Combination
Work Sheet: 07-LCR-01-WS

Introduction:

You have studied about Permutation and Combination.

Fundamental Principle of Counting and Arrangement is called **Permutation**. The number of permutations of r objects selected out of n is denoted by rnP and it is given by

$$rnP = \frac{n!}{(n-r)!}$$

Combination is a way of selecting some items from a collection, or counting the number of distinct groups formed. The number of ways of selecting r objects out of n is denoted by rnC and it is given by

$$rnC = \frac{n!}{r!(n-r)!}$$

Number of permutations with replacement:

$$rnP \ with\ replacement = n^r$$

Number of combinations with replacement:

$$rnC \ with\ replacement = rr + n - 1C = \frac{(r+n-1)!}{r!(r+n-1-r)!} = \frac{(r+n-1)!}{r!(n-1)!}$$

1. I have 4 different shirts and 3 different trousers. How many choices of outfits do I have today?
   - Whether I select shirt first or trouser, the order does not matter – I must wear shirt on top and trouser below.
   - I have 4 different ways of selecting a shirt. For each shirt, I have 3 options for trousers. Thus, there are 4 X3 = 12 different outfits possible.
   - This is an example of combination

$$14C \times 13C = \frac{4!}{1! \times 3!} \times \frac{3!}{1! \times 2!} = 4 \times 3 = 12$$

2. I have 3 varieties of green grapes and 2 varieties of purple grapes. How many types of juice can I make where each juice needs 1 green variety and 1 purple?
   - Whether I select green grape first or purple grape does not matter – they get mixed in the juice anyway.
   - Therefore, this is a case of combination
   - I have 3 options for green grape. For each option I have 2 options of purple grape
   - Therefore, total types of juices = 3 X 2 = 6
   - Or,

$$13C \times 12C = \frac{3!}{1!\times 2!} \times \frac{2!}{1!\times 1!} = 3\times 2 = 6$$

3. There are 25 children in a class. In how many ways can Teacher select a class monitor?

   Any of the 25 children can be a class monitor – there are 25 possible choices. The problem is of selection – 1 out of 25:

$$125C = \frac{25!}{1!24!} = 25$$

4. There are 25 children in a class. In how many ways can Teacher select a 2-member debating team?

   The first team member can be any of the 25 – i.e., there are 25 choices. However, after selecting the first, there will be only 24 children left for second member. Thus, for each of the 25 options for first member, there are 24 options for the second. But, order does not matter. Or, the problem is of selection – 2 out of 25:

$$225C = \frac{25!}{2!23!} = 25\times 12 = 300$$

5. There are 25 children in a class. In how many ways can Teacher can select class president and vice-president?

   How is this different from the earlier problem?

   In the earlier problem, a selection of Rita and then Arun was the same as Arun and then Rita – i.e., order of selection did not matter.

   However in this problem, Rita as president and Arun as vice-president is different from Arun as president and Rita as vice-president – i.e., order of selection is important!

   Therefore, this is a permutation problem.

$$225P = \frac{25!}{23!} = 25\times 24 = 600$$

6. There are 4 teams. If each team must play every other team, how many matches will be played in all?

   If the teams are A, B, C, D – remember that A playing against B is the same as B playing against A. So, order of selecting the 2 teams does not matter. Hence, this is a combination problem.

$$24C = \frac{4!}{2!2!} = 6$$

7. How many different 1 or 2-digit numbers can be formed using the digits 5 and 3, without repeating any digits?

   Take 1-digit numbers. We have only 2 digits 3 and 5. Hence only 2 1-digit numbers are possible.

   Take 2-digit numbers. Order of selection is important because 35 is different from 53.

   So, it is a permutation problem.

$$12P + 22P = \frac{2!}{1!} + \frac{2!}{0!} = 2 + 2 = 4$$

8. How many different 3-digit numbers can be formed using the digits 2, 4 and 7 where repeating digits are allowed?

   Order of selection is important and repetition is allowed so we use the permutation with repetition formula:

$$33P \ \textit{with replacement} = 3^3 = 27$$

9. There are 8 persons and 2 cars. Each car takes 4 persons. In how many ways can the 8 people travel?

   Order of selection is not important since the cars are not distinguished in any way. So the problem is that of selecting 4 people out of a group of 8 (remaining 4 will automatically go in the other car).

$$48C = \frac{8!}{4!4!} = \frac{8\times7\times6\times5}{4\times3\times2\times1} = 70$$

10. I must colour the hat, coat and boots of a cartoon by selecting any 3 different colours out of yellow, green, blue, red and brown. In how many ways can I colour the cartoon?

    Choosing yellow for the hat is different from choosing yellow for the coat or boots – thus, order of selection is important here. Hence, it is a permutation problem with no replacement since all 3 have to be different colours.

$$35P = \frac{5!}{2!} = 60$$

11. How many 3-digit numbers are there?

    Valid 3-digit numbers can have repeated digits. Also, they should not start with 0. Now, we know that valid 3-digit numbers are from 100 to 999 – i.e., 900 numbers in all.

    Let's try to derive this using permutation.

    Total ways of arranging 3 digits with repetition from 10 digits is

$$310P \ \textit{with replacement} = 10^3 = 1000$$

Of these, how many numbers start with 0? Say, we have 0 in the first place. Then we can select any 2 digits for the 2nd and 3rd place from 10 available digits with replacement in 10^2 ways. Therefore, we should subtract this from the total ways. So, the answer is

$$1000 - 10^2 = 1000 - 100 = 900$$

12. How many 3-digit even numbers are there?

Now from the previous example we know that there are 900 valid 3-digit numbers. Of these, half are even – i.e., 450.

Let's try to derive this using permutation.

We want numbers that do not start with 0, so first place has 9 choices, last place can be 0, 2, 4, 6 or 8 which is 5 choices and the middle place has 10 choices.

Therefore, the final answer is:

$$9 \times 10 \times 5 = 450$$