



# Graph Coloring

## Lesson Plan: Class 08 / DM / 01



Overall goal of the lesson	Understand how to color maps with minimum number of colors. The states in a map are related to vertices and their relations are defined by edges. How do we color these vertices such that two vertices connected by an edge are colored differently?
Prior knowledge required	Basic India map

**MODULE 1:**      **Module time:** 35 minutes

<b>Goal:</b>	Color the map such that neighboring states are colored differently
<b>Description:</b>	Color Maharashtra and its neighboring states with different colors such that states sharing a border have different colors. What is the minimum number of colors required? Do the same to the USA map. Understand that the states are vertices and the relation “neighbor” is an edge. The map coloring problem is translated to a graph coloring problem where vertices have to be colored differently
<b>Material required:</b>	<b>Physical: A worksheet is available with India, USA maps. Two random maps (just shapes) are available for coloring. Three sets of vertices connected by edges is provided for coloring.</b> <b>Electronic: Power point</b>
<b>Procedure Details:</b>	<ol style="list-style-type: none"><li>1. Explain that maps are colored differently so that the states can be differentiated easily. The problem to be addressed is - what is the minimum number of colors required to color the map in such a manner.</li><li>2. Explain states are entities that need to be differentiated. There is a relation among the states. Each state that shares a border is a neighboring state. Neighboring states need to be colored differently. This helps us make out different states.</li><li>3. Ask the students to color Maharashtra orange.</li><li>4. Next color Telangana, Madhya Pradesh and Goa green</li><li>5. Next color Karnataka, Chhattisgarh and Gujarat blue. Ask them what the states are, what their capitals are.</li><li>6. Mention that there is a four color theorem. In mathematics, the four color theorem, or the four color map theorem, states that, given any separation of a plane into contiguous regions, producing a figure called a map, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color. The understanding of this theorem is 4 colors are sufficient to color a map.</li><li>7. Explain that there is a 5 color theorem also. The five color theorem is a result from graph theory that given a plane separated into regions, such as a political map of the counties of a state, the regions may be colored using no more than five colors in such a way that no two adjacent regions receive the same color. So at least 5 colors are required is the inference. This is seen in the other coloring exercises.</li><li>8. Ask the students to color USA. Depending on time they can color a smaller region.</li><li>9. Tell the students that it is not required that they be states. A random map also can be colored. The next map requires 5 colors if we take the border also. It can be kept white thus making only 4 colors.</li><li>10. Make it clear to the students that regions can be connected at points. They are not neighbors.</li></ol>

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|  | <ol style="list-style-type: none"><li>11. Once the students are comfortable with the concept of coloring ask them to imagine points in the states of India. These could be the state capitals. Explain that these points are vertices.</li><li>12. Indicate that the relation neighbors can be defined as edges connecting the vertices. Multiple relations are possible. Gujrat is connected to (is a neighbor of) Maharashtra and Madhya Pradesh. Show the multiple lines and articulate the neighbor relation and edges concept.</li><li>13. Tell the students that maps are actually not required. One can just use vertices and edges. Say that this is Graph Theory.</li><li>14. In mathematics graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects.</li><li>15. Transition from a map coloring problem to a graph coloring problem with the vertices and edges. Color the vertices in a manner that two vertices connected by an edge are not having the same color. What is the minimum number of color required? The minimum needs emphasis. We can color every vertex with a different color.</li><li>16. Take the first graph. Color vertices blue. Color the first vertex and then color all other vertices not connected to this blue vertex - blue. Ensure that any new vertex you color is not connected to the other vertex with the same color.</li><li>17. Go to the next Graph.</li><li>18. In the last graph indicate that the vertex need not be circle. The shape does not matter. It is all about vertices (any shape) and lines connecting them if there is a relation.</li><li>19. Conclude this lesson with a few questions for the students. They have been coloring maps, vertices. How do we use of all this in one's life? Is it really important? Can they use graphs in planning the time table? What needs to be done in a timetable? Discuss this. Subject are in different periods on different days. Why is that? Give an example of a teacher teaching two different classes or two different subjects. Indicate that this will be discussed in the next class.</li></ol> |
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