



Mathematical Models Of Physical Biological

Lesson Plan: Class 08 / IPP / 02



Overall goal of the lesson	Using mathematical models of physical/biological systems to predict future behavior
Prior knowledge required	None

MODULE 1: **Module time:** 35 minutes

Goal:	Introduce the concept of modeling of physical/biological systems to predict future behavior
Description:	
Material required:	Physical: Printed copy of the worksheet WS-08-IPP-02 for each student, 3 graph papers pencil and eraser. Electronic: (Power Point presentation) PPT-01-IPP-02
Procedure Summary:	Go through each slide and explain the examples. Ask students to work on the examples as mentioned in the procedure. The solution to the worksheet problem is given in lesson plan.
Procedure Details:	<p>Slide#2: Ask students whether they can predict the future behaviour of a system.</p> <p>Slide#3: Let us start with an example. There is statistical data showing wind energy production for 10 years. Ask students whether they can predict the wind energy production in 2030. Ask them to plot a graph of wind energy v/s respective years.</p> <p>Slide#4: Show the students how the graph looks like. Ask them to extend the line to get wind energy in 2030.</p> <p>Slide#5: Show the students the extended graph. Ask them whether they have got the same.</p> <p>Slide#6: Now ask the students to predict wind energy in 2130 by extending graph. It is not possible to fit the graph on the graph paper. Thus tell them that we go for mathematical model of system when system has predictable behavior.</p> <p>Slide#7: Tell the students that by observation the graph is a straight line. Equation of straight line is $y=mx+c$ where m is the slope and c is y axis intercept. Now we have to calculate these constants.</p> <p>Slide#8: This slide shows how to calculate the slope (m) of a line.</p> <p>Slide#9: This slide shows how to calculate the y axis intercept (c).</p>

Slide#10: Substitute the constants in the straight line equation to get linear equation to predict wind production.

Slide#11: Substitute $x=2030$ years to get the wind energy production in 2030. This matches with our solution by extending graph. Using mathematical model we can now easily find out wind energy production in 2130 as well.

Slide#12: Explain students the steps we have followed to achieve our aim of predicting future behavior of system.

Slide#13 to Slide#18: Request you to go through the slides before reading the explanation. These slide covers one more example of mathematical modeling of lunar phases. The approach which we have derived in example 1 and described in slide#12 is followed. There is one difference in this example. This is a cyclic model thus the modeling technique used is quite different. Here we first by divide total number of days by days required for one cycle. The remainder gives total days left in the last cycle. Then follow steps in slide#17 to find the lunar phase for given date. Finally slide#18 shows how our model can predict lunar phase for 7 December 2017.

Slide#19: End of presentation

Note: If you think students will not be comfortable using x' (x prime) as a variable, you may use some other variable name (e.g, y , z , k).

Solution to worksheet problem:

Can you predict the population of Rabbits on an island in 2020?



- **Step 1 : Population data table**

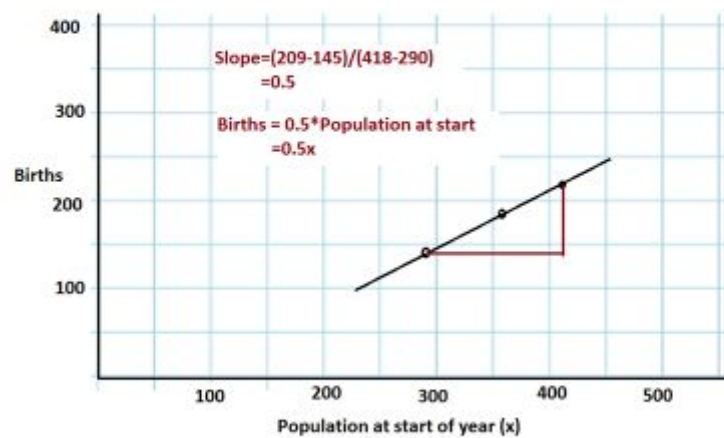
Year	Population at start of year (x)	Births in the current year	Deaths in the current year
2014	290	145	87
2015	348	174	104
2016	418	209	125

- **Step 2: Analysis of data**

Let us find births and deaths in terms of current population at start of year

Plotting Birth v/s Current population at start of year

Current population at start of year be x .



The number of births is 0.5 times the current population ($0.5x$)

Plotting Deaths v/s Current population at start of year

Current population at start of year be x .



The number of Deaths is 0.3 times the current population ($0.3x$)

- **Step 3: Mathematical modeling the system**

Let Increase in population be x' .

For every year increase in population is,

$$x' = \text{Births} - \text{Deaths}$$

$$x' = 0.5x - 0.3x$$

$$= 0.2x$$

Thus population at end of year is addition of population at start and increase in population $\rightarrow x+x'$

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- **Step 4: Predicting future behavior of system**

Population at end of 2016 is $418+(209-125)=502$.

Using the equations we can calculate predicted increase in population as follows:

Year	Current Population x	Increase in population $x' = 0.2x$	Population at end of year $x+x'$
2017	502	100	602
2018	602	120	722
2019	722	144	866

Thus at start of 2020 the population will be 866.

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