Spanning Trees
Lesson Plan: Class 07 / IP / 02

Overall goal of the lesson	**Examples of sorting, introduction to derangement**
Prior knowledge required	None

MODULE 1: **Module time:** 35 minutes

Goal:	Examples of sorting; introduction to the concept of derangement
Description:	Explain what sorting and derangement are, with the help of examples
Material required:	**Physical:** Writing material, printouts of the work sheet **Electronic:** None
Procedure Details:	1. Slide 2: We will use the example of ordering books by height as a way to introduce the problem of sorting (we will use the term in the next slide). Ask students what are the other ways to arrange books. Examples: Alphabetical order by titles; by thickness; by no. of pages etc. 2. Slide 3: Introduce Sorting. Take a simple example of ordering people by height. Use the animation to show the ordered sequence can be created 3. Slide 4: Talk about the role of comparison between elements. This is important in sorting because unless we can compare, we can't order. Ask students to point which apple is bigger, and which clock time is earlier 4. Slide 5: A brief word about sorting algorithms and the usefulness of sorting. We are not going into the details. 5. Slide 6: Run Sorting Exercise 1. The tricky part about this problem is that we want the words sorted alphabetically, and not see them as numbers (which would have a different ordering). Show the answer and explain it using the alphabetical ordering. 6. Slide 7: Run Sorting Exercise 2. There is a natural ordering for colours in the rainbow, so this shows a different kind of ordering, apart from the usual number or alphabet ordering. Either of the answers given is acceptable. 7. Slide 8: Now proceed to 'derangement'. First, we explain what disordering is, by taking an ordered sequence of apples and disturbing the order. Show the contrast between the top and bottom sequence. 8. Slide 9: Introduce 'derangements'. We will use a running example for the next few slides, with a sequence of monkeys and hats with colours. 9. Slide 10: There is a sequence of monkeys, 1-4. Each monkey has a colour. In the ordered state, each monkey has a hat of the same colour. 10. Slide 11: Now we have shuffled the order of the hats. Point out that now, none of the monkeys have a hat of the same colour as them. This is now a 'deranged' sequence. 11. Slide 12: We'll take 3 exercises to help identify deranged sequences. On this slide, the sequence is not fully deranged, because 3rd and 4th monkeys still have hats of the same colour 12. Slide 13: Here, the sequence is deranged. Ask students to confirm. 13. Slide 14: Same as Slide 13.

14. Slide 15: Now, we want to get students to think about all the many ways in which arrangements and derangements can be done. We begin with a simple example: if there is a single monkey and 4 hats of different colours, how many non-matching hats can that monkey get? The answer is three (i.e. leaving out RED)
15. Slide 16: Now, we enumerate the various possible arrangements, using the example of two monkeys and two hats. There are 2 possible sequences.
16. Slide 17: We now find out, out of these 2 sequences, if there are any derangements The one on the right is a derangement. Show this.
17. Slide 18: We move to the next level of difficulty by looking at 3 monkeys and 3 hats. Now there are 6 possible sequences
18. Slide 19: For each of the sequences, we see if they are derangements. We will find 3 derangements. In case they are not derangements, point out why (in every sequence, the hats that correspond to the monkeys of the same colour are marked with an X)
19. Slide 20: We want to point out that the number of sequences can be very large. So far, we saw sequences for 2 and 3 elements. We show the number for other numbers.
20. Slide 21: Conclude by summarizing
21. Slide 22: Hand over the worksheets

ANSWERS TO THE WORKSHEET PROBLEMS

Puzzle 1: 7, 8, 12, 16, 32, 58, 99
Puzzle 2:

Country	Canada	China	Australia	India	Argentina	Mexico	Iran
Area	9,984,670	9,572,900	7,692,024	3,287,263	2,780,400	1,964,375	1,648,000

Puzzle 3: Sequences 1 and 2 are derangements. In Sequence 3, ORANGE is in the right place and so this is not a derangement

Puzzle 4: Here are all the six possible sequences. If the sequence is not a derangement, the numbers still in position are marked by underlining them:

1. <u>10, 20, 30</u> – Not a derangement
2. <u>10</u>, 30, 20 – Not a derangement
3. 20, 10, <u>30</u> – Not a derangement
4. 20, 30, 10 – Derangement
5. 30, 10, 20 – Derangement
6. 30, <u>20</u>, 10 – Not a Derangement

Puzzle 5: The answer is **two**.
Explanation:

- Think of envelopes as being marked as E1, E2, E3 and their corresponding letters as L1, L2, L3
- There are 6 possible arrangements of letters in envelopes. List them out. See table below

- Remove any arrangements in which at least one letter is matched to its corresponding envelope. There are 4 such arrangements.
- Hence the answer is 2.

Arrangements	**Envelope 1**	**Envelope 2**	**Envelope 3**	**Arrangement in which no letter is in the right envelope?**
1	Letter 1	Letter 2	Letter 3	
2	Letter 1	Letter 3	Letter 2	
3	Letter 2	Letter 1	Letter 3	
4	**Letter 2**	**Letter 3**	**Letter 1**	**YES**
5	**Letter 3**	**Letter 1**	**Letter 2**	**YES**
6	Letter 3	Letter 2	Letter 1	

Puzzle 6: The answer is **eight**

Explanation:

- Think of envelopes as being marked as E1, E2, E3, E4 and their corresponding letters as L1, L2, L3, L4
- There are 24 possible arrangements of letters in envelopes. List them out. See table below
- Find arrangements in which exactly one letter is matched to its corresponding envelope. There are 8 such arrangements. Hence the answer is 8.

Arrangements	**Envelope 1**	**Envelope 2**	**Envelope 3**	**Envelope 4**	**Arrangement in which exactly 1 letter is in the right envelope?**
1	Letter 1	Letter 2	Letter 3	Letter 4	
2	Letter 1	Letter 2	Letter 4	Letter 3	
3	Letter 1	Letter 3	Letter 2	Letter 4	
4	**Letter 1**	Letter 4	Letter 2	Letter 3	**YES**
5	**Letter 1**	Letter 3	Letter 4	Letter 2	**YES**
6	Letter 1	Letter 4	Letter 3	Letter 2	
7	Letter 2	Letter 1	Letter 3	Letter 4	
8	Letter 2	Letter 1	Letter 4	Letter 3	
9	Letter 2	Letter 3	Letter 1	**Letter 4**	**YES**

10	Letter 2	Letter 4	Letter 1	Letter 3	
11	Letter 2	Letter 3	Letter 4	Letter 1	
12	Letter 2	Letter 4	**Letter 3**	Letter 1	**YES**
13	Letter 3	Letter 1	Letter 2	**Letter 4**	**YES**
14	Letter 3	Letter 1	Letter 4	Letter 2	
15	Letter 3	Letter 2	Letter 1	Letter 4	
16	Letter 3	Letter 4	Letter 1	Letter 2	
17	Letter 3	**Letter 2**	Letter 4	Letter 1	**YES**
18	Letter 3	Letter 4	Letter 2	Letter 1	
19	Letter 4	Letter 1	Letter 2	Letter 3	
20	Letter 4	Letter 1	**Letter 3**	Letter 2	**YES**
21	Letter 4	Letter 2	Letter 1	Letter 3	
22	Letter 4	Letter 3	Letter 1	Letter 2	
23	Letter 4	**Letter 2**	Letter 3	Letter 1	**YES**
24	Letter 4	Letter 3	Letter 2	Letter 1	

Puzzle 6: The answer is **two.**

Enumerate the arrangements as follows:

Girl 1	**Girl 2**	**Girl 3**	**Are all pairs from different classes?**
Boy 1	Boy 2	Boy 3	
Boy 1	Boy 3	Boy 2	
Boy 3	Boy 1	Boy 2	YES
Boy 3	Boy 2	Boy 1	
Boy 2	Boy 3	Boy 1	YES
Boy 2	Boy 1	Boy 3	