Spanning Trees
Lesson Plan: Class 07 / DM / 01

Overall goal of the lesson	**Introduction to Spanning Trees**
Prior knowledge required	None

MODULE 1: **Module time:** 35 minutes

Goal:	Introduction to spanning trees
Description:	Learn what spanning trees are and its practical applications
Material required:	**Physical:** Writing material, printouts of the worksheet **Electronic:** None
Procedure Details:	1. Slide 2: We start with an extremely basic example, which we call Puzzle 0. With this,we will illustrate a simple loop and how to make a tree out of it. This is a 3 node diagram with 3 connections (edges) between them. Point out the loop. Now ask: how can we remove one connection such that the loop is broken? Any of the three can be removed. Use the animation to show the answer. NOTE: the usual computer science term for the dots are 'nodes' or 'vertices'. To simplify, we are using the word 'dot'. Similarly, instead of 'edges', we are using connections. All the puzzle diagrams are also given in the worksheet. 2. Slide 3: Puzzle 1 We will take a slightly complex example. This has 5 dots and 6 connections in all. Ask 1-2 students to point out the loops. Ask them to repeat the same exercise as in Puzzle 0: how to remove loops by removing the connections? Also point out that when they remove connections, no dot should be 'alone' i.e. connection-less. On the next slide, we will give 4 options but let students think about this for a minute before we go to the next slide. 3. Slide 4: There are 4 solutions here, of which some are correct solutions. One by one, show each of the options. Ask students to consider each options, by thinking about: a. Is there a loop? b. Are any of the nodes alone? 4. Slide 5: Show the answers. a. is incorrect, as there is a loop (shown by the dotted line) b. There is no loop but it is also incorrect, as the top right dot has no connections c. This is a valid solution. Since two connections are criss-crossing, there may be some confusion about this being a loop. Point out this is not a loop since there is no dot at the intersection. d. This is also a valid solution 5. Slide 6: Let's explain the solution further and learn an important term i.e. 'trees'. a. Trees are diagrams without loops. They are called trees because they look like they are branches and leaves from a root. b. When seen from the left, the 2nd solution looks like a tree, starting from a root

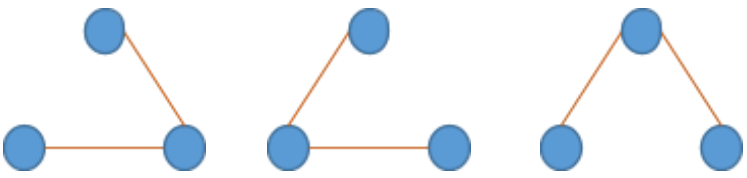
6. Slide 7: We now explain what a Spanning Tree is. We recap the starting diagram for Puzzle 1, and show the two solutions. These are called Spanning Trees. Read out all the statements.
   They are trees because they have no loops.
   They are called Spanning trees because they span all the original dots.
7. Slide 8: Puzzle 2: Here's a puzzle for the students to solve on their own. There are 6 dots and 9 connections. Ask the students to draw one or more spanning trees.
   Recap that:
   - There should be no loops in the solution
   - All the dots should be connected with at least one connection
8. Slide 9: Show a couple of valid solutions. There could be others as well.
9. Slide 10: we now look at a practical application of Spanning Trees. We'll take an example of building roads. Here's a map of 5 cities – each dot is a city.
10. Slide 11: Now, we have to connect these cities so that we can go from any city to any other city.
    The obvious solution is to build a road from every city to every other city. By building a total of 10 roads, we can do this.
    However: this is a 'perfect' solution, but in reality, it is tough to build so many roads. Is there a way to connect them with fewer roads?
    Here's where Spanning Trees help.
11. Slide 12: Spanning Trees help find a smaller number of roads because
    a. There are no loops (so you have only the minimum number of roads)
    b. All cities are connected – you may not be able to go directly from one city to another. But you can find some route from any city to any other.
    c. No cities are left out alone

    Here's an example of a spanning tree for this map. There are 4 connections that link all
    the 5 cities.
    Ask students: can you find a route from Chennai to Delhi? (the answer is: Chennai – Ranchi – Pune – Delhi)
12. Recap of what we learned.
    - We started with a diagram with dots and connections, some of which had loops
    - We removed them to get a 'tree'. A 'tree' does not have loops
    - A spanning tree is a type of tree based on an original diagram which connects all the dots but has removed all the loops in the original diagram
    - An application of spanning trees is to design minimum no. of roads between cities without spending too much money
    - In the next lesson, we'll look at advanced versions of spanning trees

ANSWERS to the WORK SHEET PROBLEMS

Puzzle 1

Any solution with one line removed is correct. i.e.



Puzzle 2

Correct solutions are: (a) and (c).
(b) is incorrect because the rightmost dot is not connected
(d) is wrong because there still is a loop

Puzzle 3

There are many possible solutions. To check if a solution is correct, do the following:

a. Are all dots connected? i.e. there is at least one line that connects every dot.
b. Do you see any loops anywhere?
c. Check that no new lines or dots have been added

Here are some of the possible correct solutions: